On the Determination of Heights by Means of the Barometer. Part I. continued. By W. Mathews, Jun., M.A.

In our last number Laplace's formula for determining heights was explained at length; but, as was there mentioned, the hypothesis respecting the temperature assumed by him requires further investigation. We shall now proceed to examine this hypothesis, and to enquire how far it is in accordance with fact.

The coefficient of dilatation of dry air for each degree Cent., under a constant pressure, as determined by Gay Lussac, is ·00375, or $\frac{1}{270}$th, very nearly, of its volume at 0°. Suppose this law to hold good for all diminutions of temperature beyond those for which it has been possible to verify it in practice, and a mass of air which occupied a space of 1 cubic foot at 0° to be cooled down by successive reductions of temperature of 1° Cent., the pressure remaining the same, at each reduction the volume would be diminished by the $\frac{1}{270}$th of a cubic foot, and when the temperature of $-269°$ Cent. had been reached the next step to $-270°$ would annihilate the volume, and render the density infinite. At this point the formula becomes physically impossible. If so low a temperature could be produced it would probably correspond to a sudden change in the physical constitution of the mass, unless, which is not improbable, such a change took place before that temperature was reached.

The temperature $-\dfrac{1}{\cdot 00375}$ or $-270°$ Cent. is termed the absolute zero, and temperatures measured from it are termed absolute temperatures.

Now the law assumed by Laplace, which leads to the relation $m=\frac{1}{2}(s+t)$, is that *the differences of the squares of the absolute temperatures are proportional to the differences of height.*

Suppose the temperature at the sea level to be 20° Cent., or 290° above the absolute zero. The increments of height x_1, x_2, x_3, &c., corresponding to decrements of 1° in the temperature, will be given by the equations—

$$\begin{aligned}\mu x_1 &= 290^2 - 289^2 \\ &= 579 \\ \mu x_2 &= 577 \\ \mu x_3 &= 575, \text{ \&c.}\end{aligned}$$

where μ is a constant ratio, which must be determined by observation.

The height above the sea level which actually corresponds to a decrement of temperature of 1° Cent. is not very different from 579 feet, and μ may therefore be roughly taken as equal to unity, and the right-hand members of the equations as representing feet.

The increments of height, therefore, expressed in feet, corresponding to successive decrements of 1° Cent., will form the series—

579, 577, 575, 573, 571, &c.

The decrements of temperature corresponding to successive increments of 1,000 feet in height can easily be ascertained. Calculated in decimals of a degree Cent. to the second decimal place, the series for the first 10,000 feet is—

1·73, 1·74, 1·75, 1·76, 1·77, 1·78, 1·80, 1·81, 1·82, 1·83:

the total decrease being approximately 17·79 Cent.

These results may be summed up as follows:—

According to Laplace's hypothesis, the temperatures decrease more rapidly in ascending, equal increments of height corresponding to continually increasing decrements of temperature. Conversely, equal decrements of temperature of 1° Cent. correspond to increments of height, decreasing in arithmetical progression, and having a common difference of about two feet.

This law of decrease is not in accordance with the generally received opinions of meteorologists, some of whom have maintained the principle of a uniform decrease, while others have considered it more probable that the temperature decreases less rapidly in ascending from the surface of the earth. The question has happily been set at rest for ever by the magnificent series of balloon observations recently made by Mr. Glaisher,* who has established the truth of the latter opinion, and proved conclusively that equal decrements of temperature correspond to continually increasing increments of height.

If the decrement of temperature of 17·79 Cent., obtained above on Laplace's hypothesis at the height of 10,000 feet, were produced at the same altitude by the law of uniform decrease, the decrement for each 1,000 feet would be 1·779 Cent. The following table exhibits the corresponding temperatures on each hypothesis:—

* *British Association Reports*, 1862–3.

Heights Feet	Temperatures on Laplace's hypothesis Cent.	Temperatures on hypothesis of uniform decrease Cent.
0	20·00	20·00
1,000	18·27	18·22
2,000	16·53	16·44
3,000	14·78	14·66
4,000	13·02	12·88
5,000	11·25	11·10
6,000	9·47	9·33
7,000	7·67	7·55
8,000	5·86	5·77
9,000	4·04	3·99
10,000	2·21	2·21

If therefore the temperatures of the extremities of an aerial column are given, the temperature at every other point of the column is greater on Laplace's hypothesis than on that of a uniform decrease, and therefore the mean temperature of the column must also be greater, and still greater than its true value according to the actual law.

The mean temperature, therefore, adopted by Laplace, produces too large a correction, and brings out the results too high; but the error is no doubt in a great measure counterbalanced by the empirical determination of the barometric coefficient, which is less than it would be if Regnault's constant were introduced.

It only remains to consider to what extent, and under what circumstances, the true mean temperature of an aerial column differs from the semi-sum of the temperatures of its extremities.

This problem may be solved in precisely the same manner as that employed by Laplace in determining the barometric coefficient, viz. by measuring trigonometrically a number of heights, and forming the corresponding barometric equations, in which the mean temperature is treated as an unknown quantity; but it is not desirable that a barometric coefficient already determined empirically should be involved in the equations. This subject has been most ably investigated by Professor Plantamour.* He ascertained by means of the spirit level the exact difference in height between the cisterns of the barometers at the Geneva Observatory, and the St. Bernard Hospice, and made a very large number of simultaneous observations at the two stations, at every season of the year, and every

* *Mesures Hypsométriques dans les Alpes.* Génève, 1860.

hour of the day and night. The tables employed were those described above, founded on Bessel's formula, which have the merit of being free from empirical coefficients. The results are very interesting. The semi-sum of the extreme temperatures was found to be equal to the true mean two hours after sunrise and at the time of sunset, to be in excess of it in the day-time, and in defect at night. In the summer months the excesses are greater than the defects, in the winter months they are less. In March and September they are nearly equal. The defect is a maximum from 2 to 4 A.M.; the excess is a maximum from noon to 2 P.M., and at noon in the month of July amounts to 3°·62 Cent., a very considerable quantity.

The phenomenon admits of an easy explanation. In the middle of the day the radiation from the ground communicates heat to the air immediately in contact with it, much more rapidly than it can be conveyed upwards; and the observed temperature will be greater than it would be if the law of increase in descending through the superincumbent strata obtained also near the surface of the earth. An opposite effect will be produced at night. This inequality is termed the Horary Equation.

These results, though highly important, afford no evidence of the error introduced by the adoption of Laplace's law of decrease instead of the true law, that error being entirely masked by the effect of the Horary Equation. As, however, the total annual effect of this inequality must be nothing, its influence may be eliminated by taking the mean temperature of the year as the basis of the calculation. The mean temperature of Geneva for the eighteen years from 1841 to 1858 was 8°·91 Cent., and that of St. Bernard −2°·02, their semi-sum being 3°·45 Cent., while the true mean temperature of the intervening column of air, due to the mean barometric pressures for the same period, is found to be 4°·05 Cent., or 0°·6 in excess of the semi-sum.

A precisely opposite result might have been anticipated, and the error we are in search of must therefore be again overpowered by another unexpected inequality. This is explained by Professor Plantamour to arise from the mean temperature of Geneva being lowered by the effect of the glacier water in the lake. If no similar error exists at St. Bernard, the mean annual depression of temperature thus produced at Geneva must be greater than 1°·2 Cent. The amount by which the temperature of a locality is made to deviate from that due to the latitude and elevation may appropriately be termed the *Local Equation.*

It thus appears that the difference between the true mean temperature of an aerial column, and the semi-sum of the temperatures of its extremities, is, generally speaking, a variable and complex quantity, into which the following elements must enter:—

1. The error due to Laplace's hypothesis.
2. The Local Equations at the upper and lower stations.
3. The Horary Equations at the upper and lower stations.

In the case of Geneva and St. Bernard, Professor Plantamour makes the arbitrary assumption that two-thirds of the total difference are due to the former place and one-third to the latter. It is scarcely possible that this assumption should be true at all hours of the day, and in all seasons of the year, and the question deserves further investigation, as the proportion to be assigned to each station is precisely what, in practice, it is important to know. The different sources of error are, however, so involved that it will be very difficult to disentangle them. There is still, therefore, something to be done in this branch of the subject, and until the Local and Horary Equations have been determined for all the principal continental observatories, the theory of barometric calculation will hardly have been placed on a perfectly satisfactory footing.

Ascent of the Rothhorn. By the Rev. Leslie Stephen, M.A. Read before the Alpine Club on April 4, 1865.

THE little village of Zinal lies, as I need hardly inform my hearers, deep in the recesses of the Pennine chain. Some time in the middle ages (I speak on the indisputable authority of Murray) its inhabitants were converted to Christianity by the efforts of a bishop of Sion. From that time till the year 1864 I know little of its history, with the exception of two facts—one, that till lately the natives used holes in their tables as a substitute for plates, each member of the family depositing promiscuously his share of the family meals in his own particular cavity—the other, that a German traveller was murdered between there and Evolena, in 1863. Undeterred by these warnings, Grove, Macdonald, and I, with our guides Melchior and Jacob Anderegg, arrived at M. Epinay's hospitable inn in August 1864, and I am inclined to think that our arrival rather more than doubled the resident population. M. Epinay's inn, I may remark, is worthy of the highest praise. It is true that the accommodation is limited. M. and G. had to sleep in two cupboards opening